Theoretically Informed Intonation

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MHS581 - Analysis and Performance
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— Introduction —

The past 50 years of musicological research have yielded surprising discoveries and exciting paradigm shifts for performers and thinkers alike on the subject of intellectually informed performance practice. Most notable is the movement often referred to as “HIP” or “historically informed performance.” Although the official goal of HIP is to evoke what the composer “heard” in his own time, either in the flesh or in the imagination, the underlying motivation of an HIP researcher or performer is to evoke the ultimate performance of a given work, hoping that a pilgrimage to its original source will yield the best result.

Curiously absent from this noble quest, however, is detailed analysis on how intonation for flexible-pitch instruments may correlate to the harmonic and melodic organization of a particular work. Research on the tuning of keyboard and other fixed-pitch instruments may provide a baseline for how intonation was generally approached in a particular composer’s lifetime, but we cannot assume that the same practices would have been in effect for a string quartet, for example, a genre which is not bounded by the practical tuning limitations of a keyboard. Music theory and intonation both focus on the distances between notes in pitch space, and I posit that an effective performance of any piece will intimate this link. In both fields, choices must be made which deal with ambiguities presented by the composer, or by the multiplicity of meaning inherent in any work of an art experience as abstract and ephemeral as music. The quality and informedness of the intonation choices made will partially determine the degree of unity, complexity, and intensity conveyed in a given performance.[1]

The humble aim of this paper is to take an early attempt at illuminating the specific connections between theory interpretations and intonation choices. After the development of an approach to theoretically informed intonation, I will apply the approach to the opening fugue of Beethoven’s *String Quartet No. 14 in C# minor*, Op. 131 as a case study, and supplement the paper with a computerized MIDI realization of my analysis and intonation theory in action.

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[1] Monroe Beardsley, *Aesthetics: Problems in the Philosophy of Criticism* (1958), 456.

The primary fork in the road for both theory and intonation is that of harmony versus melody. This central divergence is amply reflected in any cursory review of the past century of theory textbooks, from Schoenberg to Schacter, all the way down to a choice as simple as whether to teach scales (melodic basis) or intervals (harmonic basis) first. It is still not commonly recognized, however, that an intonation decision conveys the same choice. The specifics of why this works will be discussed here with regard to general acoustics and historical approaches to intonation problems.

Acoustically speaking, the more common tunings are derived from the overtone series. The overtone series is derived from the multiples of any given frequency in Hertz (or vibrations per second): A440, A880, E1320, A1760, C#2180, and so on. This is different from octave generation, which is derived from multiplication by two: A440, A880, A1760, A3520, and so on. The result of the overtone series' additive property is the progressive decrease in musically perceived intervals as it progresses upwards (440 becomes a smaller and smaller numerical proportion in larger numbers): the first interval is an octave, then a P5, a P4, a M3, a m3, and so on to ever smaller intervals. The earlier that a given pitch relationship occurs in the overtone series, the more consonant it has been traditionally perceived to be.

Of critical importance to the relationship between music theory and intonation is the generation of a third "voice" via the difference between any two simultaneously sounding tones. This third tone is often called either a "resultant tone," a "difference tone," or a "subtraction tone" (because it is the subtractive difference between the two other frequencies simultaneously sounding), and it is not clear if it is a true physical occurrence or simply a human psychoacoustic effect. If an A880 and its octave A1760 sound simultaneously, the result is the perception of their subtracted frequency A880: this results in the strong, smooth sound of an octave, one which literally reinforces the lower octave as doubled. If an A1760 sounds simultaneously with a C#2180, their difference tone is A440, which will be clearly perceptible as real pitch if the first two are so perfectly tuned (note that, for the subtraction tone to be audible, the difference between the two other pitches must be large enough numerically to be within the range of human hearing). The especially interesting applications occur with the somewhat more exotic intervals, such as the minor third: if a C#2180 and an E2620 are so tuned, the result is a clearly audible A440 two octaves below, invoking true harmony, a full triad from just two notes. The mistuning any upper interval, therefore, is destructive to relationships with bass pitches of a work, deeply upsetting the likes of Rameau.

The twist presents itself in following problem: the overtone series does not react well to modulation. A keyboard player would be therefore ill-advised to tune his instrument in such a fashion as the intonation would only be palatable in two keys (one major and its relative minor). The history of keyboard tuning is a story of compromises, either avoiding or exploiting the imbalances created by tuning more optimally to one key. Naturally, as more keys began to be used with works such as Bach's *Well-Tempered Clavier*, more mitigated intonation schemes (such as well-temperaments and eventually equal temperament) were applied to keyboards. It does not follow, however, that string instruments always followed the same procedures, since they have never been limited in the same sense: when the key of a work switches, they can readjust the placement of all pitches to reflect the new key. As Barbieri notes, however, "the problems are multiplied when one deals with instruments of free intonation." How to do this in a way informed my music theory precepts is the special interest of this paper.

With "just" or "Syntonic" tuning, the musical scale is derived from the pure acoustic presentation of these overtones. For example, if we count the fundamental sounding pitch as a kind of first overtone, then the octave is defined as the musical distance from the second overtone to the first, the perfect fifth defined as the distance from the third overtone to the second (G to C with a C fundamental), the perfect fourth defined as the distance from the fourth overtone to the third (C down to G), the major third defined as the distance between the fifth overtone and the fourth (E down to C), and so on up and through the overtone series. These produce the ratios of 2:1, 3:2, 4:3, and 5:4 respectively, which are convenient numerical representations of musical intervals. It should be mentioned here that there is a sense that interval consonance can be defined as interval ratio simplicity, in which case the above ratios are among the most consonant. Across enough overtones we could eventually derive a complete major or even chromatic scale, keeping in mind that some of the fractions would be quite complex and therefore less digestible to the ear. The following diagrams by composer Ben Johnston will illustrate this principle:

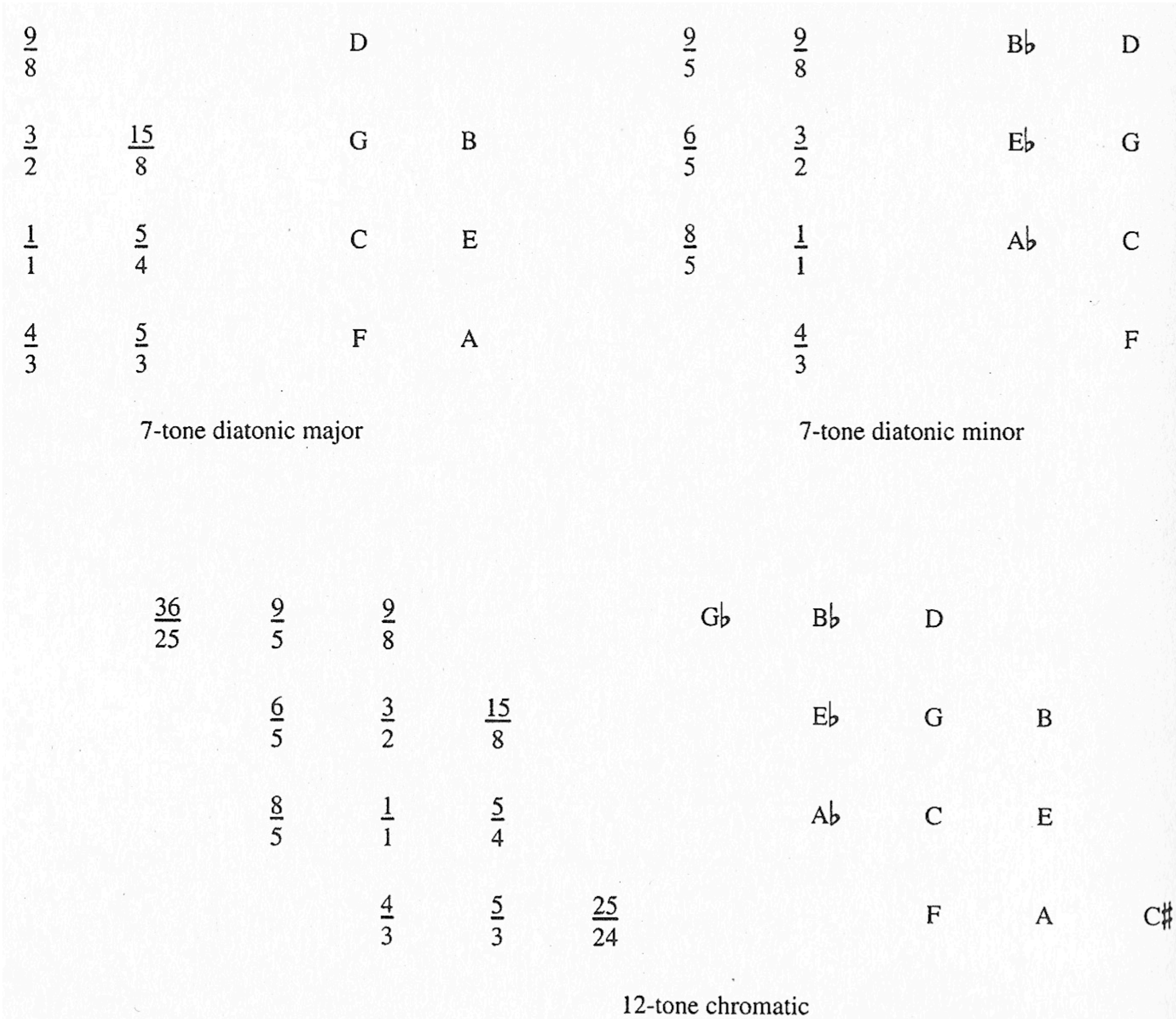

Figure 1: Lattice representations of diatonic major, minor, and chromatic scales of traditional triadic tonality. Three-limit relationships (i.e., by chains of perfect fifths) are mapped on the vertical axes, 5-limit relationships (.e. by chains of major thirds) on the horizontal axes. [2]

Historically, just intonation (or approximation) was commonly employed by free tuning instruments such as the violin. Giuseppe Tartini, in particular, was an important

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[2] Ben Johnston and Bob Gilmore, “Tonality Regained,” in *"Maximum Clarity" and Other Writings on Music* (2006), 48.

violin virtuoso and champion of Syntonic intonation in the mid-18th century. His method was so centered on Syntonic tuning that the use of open-strings was denigrated due to the Pythagorean intervals that result.[3] Francesco Galeazzi, a professional violinist of the same period in Rome, “claims that the best performers applied the Syntonic tuning in the strictest fashion, shifting the major and minor tones according to the key.”[4]

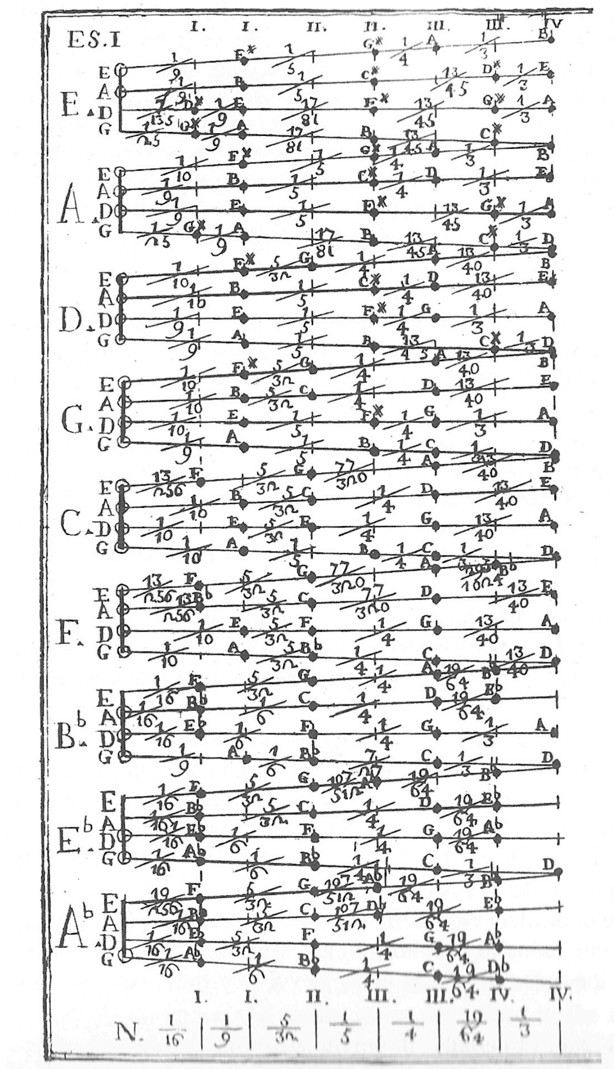

Figure 2: The above chart by Galeazzi (1791) demonstrates the thorough approach to Syntonic performance practice of the late 18th century, with full awareness of the mathematical interval proportions at work.

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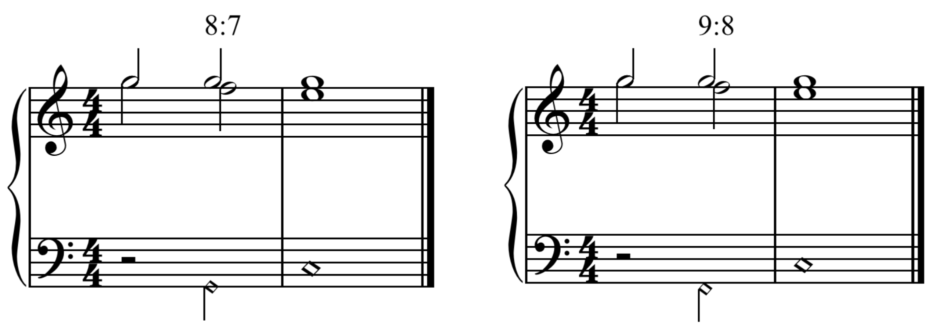
[3] Patrizio Barbieri, “Chapter C: Violin and woodwinds intonation” in *Enharmonic Instruments and Music 1470-1900* (2008), 112.

[4] Ibid., 114.

Several peculiarities present themselves with Syntonic intonation: first, the tritone and the minor seventh occur in the overtone series prior to the major seventh or either of the sixths.[5] The more convenient approach to "finding" these tones, therefore, is to derive them from the simpler ratios mentioned above: the distance between the leading tone and the tonic can be equated to the distance between the already discovered major third and perfect fourth, and the sixths can be derived as major/minor third inversions *down* away from tonic. Johnston's lattice demonstrates these interconnections well, showing that any interval motion in one direction (up, down, or diagonal) can yield a corresponding interval in the opposite direction. In the case of seconds, they can easily be of two different distances: if the interval is *functioning* as a true second then it is tuned narrower (9:8), and if it functions as a lowered seventh (against a tonic, for example), then it is tuned wider (8:7). The correct tuning of these intervals according to the above principles will yield a bass resultant tone which is serves the harmonic function of that vertical instance.



Correct **Incorrect**

Figure 3: The resultant tones are shown in diamond noteheads, which will sound from the upper notes tuned according to their respective ratios. The chordal seventh in the first example will tune noticeably lower than that in the second example. The minor third at the end in both cases is considered to be a standard 6:5 ratio. Note the clear establishment of a 3-voice cadential texture with the resultant tone.

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[5] While Johnston reflects the latter half of this fact in his lattice (showing the simple fractional ratio 9:5 for Bb), for some reason he chooses a later instance of the tritone's occurrence in the overtone series: the first true instance would be 11:8. Other instances can be interpolated or approximated at 7:5, 10:7, 13:9, and 16:11, but Johnston highlights 36:25, a much later instance and one bearing intervallic difference to 11:8 of only -2:75.

The second peculiarity is that the distance between a justly tuned major third (-13.7 cents, relative to equal temperament) and a justly tuned perfect fourth (-2 cents, relative to ET) is wide enough to be *melodically* unwieldy. The effect is particularly pronounced because scale degree 4 (especially in a major key) is a melodic “tendency tone” with a strong tonal gravitation towards scale degree 3, and the slightly larger melodic distance mitigates the attraction of the two notes. This voice-leading issue, one which applies to other melodic instances as well, presents a major problem for the actual application of just intonation. The problem transcends even the freedom of modulation in tuning offered by string instruments: one cannot avoid the melodically awkward intervals with just intonation alone, and therefore an alternate approach is necessary.

In music theory there lies an elegant balance in the analysis of harmonic versus melodic elements, and we should search for the same sophistication in our approach to intonation. While the just intonation system works magnificently for harmonic tuning, it leaves melody out to dry. The Pythagorean system works well for melodic tuning, especially as a supplemental approach.[6] In Pythagorean tuning, sequential fifths are tuned pure and the products are interpolated within one octave to provide a complete chromatic scale.[7] However, because a justly tuned perfect fifth is about 2 cents sharp (or wide), each step along the circle of fifths increases the distance of that pitch from its equal temperament counterpart. Curiously, this process means that Pythagorean tuning bears *opposite* tendencies to just tuning. For example, the major seventh in just tuning is 11.7 cents low whereas the Pythagorean major seventh is 10 cents high (C-G-D-A-E-B is 5 fifths, multiplied by 2 cents for each fifth), placing it closer to the tonic and intensifying the “leading” aspect of the leading tone. A similar melodic intimacy falls out with regard to scale degrees 3 and 4 in major keys: the Pythagorean major third is almost 22 cents higher than the Syntonic third, and the perfect fourth similarly low by 2 cents. In the late 18th century, the growing exploitation of the leading tone’s attraction to

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[6] Barbieri, 170.

[7] A Johnston-style lattice of the Pythagorean system would bear exact similarity between horizontal and vertical axes from the origin (C), expanding on either axis by the same distance of 502 cents.

tonic—as well as of scale degree b7 to scale degree 6—caused the succession of chromatic intervals' intonation to swap places, a Bb now resting *lower* than an A#.[8] A certain regard for *pathos* began to take over in practical performance, much to the simultaneous advocacy and befuddlement of Syntonic tuning champions like Galeazzi:

> "Concerning the seventh there is one more observation to be made, and it is that when it rises by one step to the octave it must always be made somewhat altered and shard so that it sounds pleasing and satisfying to the ear; which is very difficult to explain, since if in practice the interval between the seventh and octave is a minimal semitone, in theory it is a major semitone, nor do I know how the one can be reconciled with the other."[9]

The use of a Pythagorean scale degree 7 in a dominant chord at a cadence would sacrifice the harmonic purity of the dominant but lead more strongly to the tonic chord. The sacrifice of purity here is not necessary a negative: the tonal hierarchy of the dominant and tonic chords could be differentiated by the particular intonation quality of each chord (a Pythagorean dominant chord leading melodically to an acoustically pure and stable tonic triad, justly tuned). This distinction falls right in step with commonplace functional theory thinking: a perfect authentic cadence in the subdominant or other key area will not carry the same strength, weight, or finality as a similar cadence in the home key, though they may be exactly the same otherwise. A perfect fourth may be melodically consonant, but harmonically dissonant. In both theory analysis and intonation, the central struggle is the same: reconciliation of harmonic priorities with melodic ones.

So far I have hinted at a composite approach to intonation of 19th century tonal music, combining properties of Syntonic and Pythagorean tuning. Such a composite approach seems like it may have been a practical reality of mid-19th century performance practice: cello pedagogue Bernhard Romberg published a method book in 1840, noting that, "There are cases where theory and musical feeling come into

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[8] Barbieri, 157.

[9] Ibid., 162.

conflict."[10] The changing aesthetics of what we now call the Romantic Era seemed to be influencing musicians' approach to tuning, struggling to reconcile the theoretical euphoria of Enlightenment Era thinking with a new fashion for dramatically cast human emotion in art music. Janina Fyk eloquently sums up the issue:

> "The two contrasting models of intonation—static and dynamic—may be interpreted as an outcome of a specific union between the theory of music and the theory of aesthetic experience observed in a particular period. In each, aesthetic criteria characteristic of a given historical period serve as the essential structural standards. These criteria have always, however distant the past, corresponded to the prevailing views concerning the question of content in music. Transition from the static to the dynamic model of intonation was accompanied, and in some cases even preceded, by a change in aesthetic criteria combined with the emergence of a new approach to the question of content or meaning in music."[11]

In many cases the performer would theorize about intonation in one light and perform in another. The conflict in tuning approach did not go unnoticed by scientists of the period either: in separate studies on the preference of listeners for Syntonic or Pythagorean choices, experiments in 1869 (Cornu and Mercadier) and 1870 (Helmholtz and Gueroult) produced opposite results, leaving the issue murky even at the onset of the 20th century.[12]

With a "Pytha-Syntonic" composite tuning theory, there is new ambiguity about how exactly to deal with key modulation: does the new tonic occupy the Syntonic or Pythagorean position of the preceding key? Or perhaps of the home key? There are at least two important priorities to address: analytical or functional reflection, and *affekt*. In the case of the former, the solution must somehow provide for differentiation between keys, a sense of the home key as special and hierarchically superior to other keys. This requires an aspect of static tuning approach. With the latter, the emotions, gravities, and

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[10] Ibid., 165

[11] Janina Fyk, *Melodic Intonation, Psychoacoustics, and the Violin* (1995), 18-9.

[12] Barbieri, 170.

sympathies derived from certain melodic intervals must carry appropriate inflections in tuning from a purely affective point of view. Somehow, the tuning system must also be dynamic, to reflect the dynamic state of the human condition.

Because the purity, smoothness, and natural consonance of the Syntonic tuning system suggests a state of rest or stasis, I believe it should be the basis of a well-developed and practical tuning system for 19th century tonal music. Modulation to even adventurous key areas merely suggests a new Syntonic center: the pitch relationships will be similar to the home key and therefore harmonious, but the Syntonic centering of that new key in pitch space will subtly reinforce the overall centricity of the prime key area. The following lattice by Ben Johnston will help to visualize the multiple possible centricities in Syntonic pitch space, while also demonstrating that such a system still has an overall centricity towards its simpler intervallic ratios:

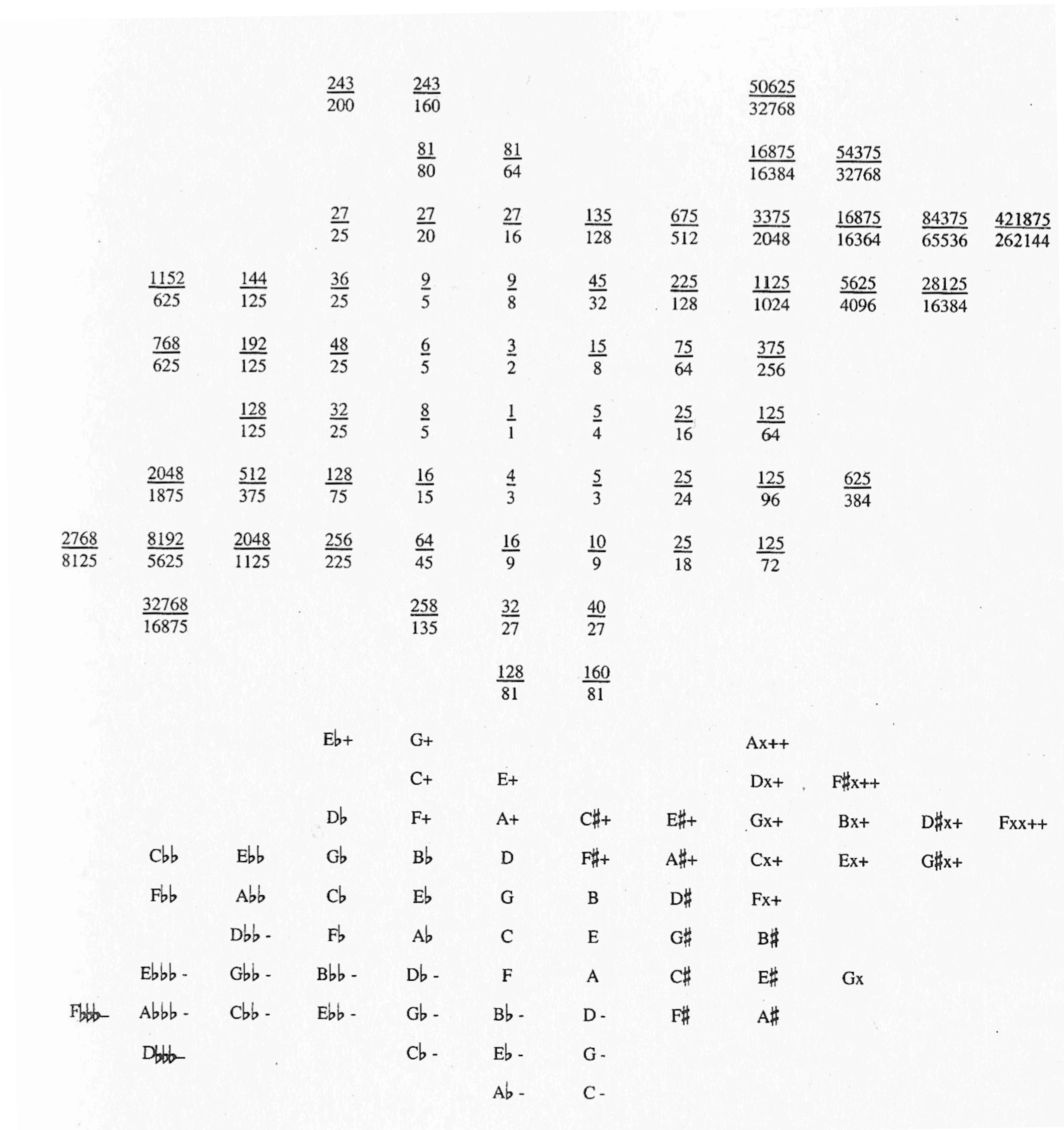

Figure 4: Lattice of the 65-tone hyperchromatic triadic system (using prime numbers 2, 3, and 5), expressed in ratios (above) and in letter names (below).[13]

[13] Johnston, 49.

— APPLYING A THEORY OF INTONATION: BEETHOVEN, OP. 131 —

The backdrop of Johnston's "hyperchromatic triadic system" will provide a reference pitch system built on consonance and intervallic purity, but one which will be inflected by Pythagorean tunings where melodic tension is of special significance. It is a dynamic approach to tuning, as defined by Fyk.[14] Here are some rules for how those inflections will be carried out in an organized and replicable manner:

1. Chordal tendency tones will be tuned in the direction of their melodic attraction.
2. The same tones, when inflected in Pythagorean tuning, will not drag other simultaneously sounding tones out of Syntonic tuning except where perfect intervals are affected: in this case, other tones related by fifth and fourth are also acting as tendency tones and must be correspondingly inflected.
3. Certain chords may have strong contrapuntal roles, in which case their roots may act as tendency tones under rules 1 and 2
4. The tonic chord in every key will be Syntonically pure
5. The tonic pitch of alternate keys will be grounded in the Syntonic pitch space of the home key.
6. Non-chord tones will usually be tuned Pythagorean as contrapuntal gestures.
7. In the case of a *legate* canonic or real fugal treatment, an attempt will be made to maintain the specific intonation of the opening *dux* in future iterations.
8. Spontaneous accommodation can be made to reflect changes in mood evidenced by non-metric or -pitch related elements (such as dynamic indication).
9. Where possible, the purity of perfect fifths and fourths will be maintained.

In the case of the fugal opening from Beethoven's *String Quartet No. 14 in C# Minor*, Op. 131, we are presented with many tuning problems which are intimately related to analysis and interpretation. The first problem is the presence of a canonic or fugal element which is imitated at secondary transpositions. The second problem is that

[14] Fyk, 18.

this particular fugal subject is something of a polyphonic melody, invoking both harmonic as well as harmonic implications:

Figure 5: The fugal subject which opens the quartet.

The anacrusis G# connects more to the A in the second bar than it does to the following B#. What results is two pairs of ascending half-steps: B#-C# and G#-A. While the former creates tonal resolution, it is interpolated between the latter, which creates impassioned musical tension. The active upper half of a C# harmonic minor scale (G#-A-B#-C#) has here been split up into its harmonic dyads, separating the augmented 2nd while evoking the melodic drama resulting from the divergent tendency tones B# and A. The musical tension is dissipated via conjunct motion in the lower and more stable half of the minor scale, decorating scale degree 5. The B# in measure 1 and the A in measure 2 should be inflected in Pythagorean tuning, the first being raised and the second lowered. They are tendency tones which are further expressed by their strong metric placement and dynamic indication, and they therefore fall under rule #1 to be tuned in the direction of their melodic attraction.

The sudden change in character going into the third bar suggests, however, a different mood and therefore a different tuning of the next A natural: under the force of the sforzando, first A was tuned low, but in light of the new more stable and regular character a less expressive Syntonic A (+14 cents) will better serve the aesthetic of this particular subject. In the case of the third scale degree E in the fourth measure, the Syntonic treatment here serves both melodic as well as harmonic functions: the implied harmony here would be a tonic chord (if only briefly), and a Syntonic E (+16 cents) would sound harmonically pure in this context, as well as leading back up out of its neighbor motive. Furthermore, the perfect fourth connection between A in measure 3 and E in measure 4 demands that they be tuned within the same system, acting as melodic boundary points for the second half of the subject. The F#s, G#s, and C# are all easy decisions because their adjustments are the same in both tuning approaches (-2c, +2c, and ±0c, respectively).

In the case of the next fugal entry, which is oddly in the subdominant, all Syntonic tonal centricity reorients around F# with analogous Pythagorean inflections on E# and D natural.

Figure 6: Violin 2 entrance, fugal real answer at the subdominant.

The C# in the first answer will tune in exactly the same location as before, even in the new key context. The A natural will tune Syntonically and in the same place as the A from measure 3 (*not* the same as the A from measure 2). The B natural will tune Syntonically in the new key, but will have a different pitch position than B natural in the original key (a ratio of 16:9 instead of 9:5). To illustrate why this is so, it is helpful to again consult Johnston's pitch lattice in Figure 4. Notice that if one were to map the subject onto the origin (1:1) and then draw the same design centered around the subdominant (4:3), there would be many but not all overlapped pitches. The 9:5 B natural (or scale degree flat-7) would be accessible from 1:1, but it is a distant pitch relationship to 4:3 as a center and therefore would not serve the same harmonic or melodic function.

In the countersubject, we begin to see our first signs of real chromatic opportunity, a chance to evoke the power and artistic potential of a hybrid dynamic intonation system:

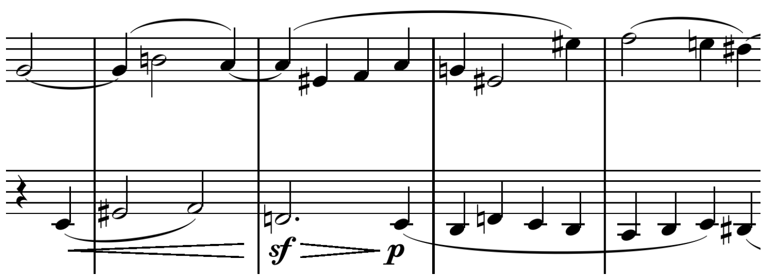

Figure 7: Violins 1 and 2, treble clef, fugal answer and countersubject.

The B natural in the upper voice is scale degree 4 in the new key and therefore should sound as the same aforementioned 16:9 interval (subdominant *of* the subdominant 4:3). The following A natural should be tuned Syntonically, so that the Pythagorean D natural in the second violin will be as surprising, abrasive, and expressive as it needs to be, further accenting the note's "sforzando." The E# and F of the countersubject echo the same gesture in the answer and should be tuned exactly the same. Interpretation of the second and third E#s, however, is thrown into question by the b2, G natural. Beethoven here is playing with enharmonic ambiguity: the chord progression in that bar is essentially Neapolitan bII6 - V4/2. The issue arises in how to interpret the metrically syncopated E# in the middle of the measure: as an anticipatory leading tone, or as an enharmonically respelled chordal seventh? Because the following E# is clearly a leading tone, the most compelling and practical solution might be to treat this tone as an anticipation not intended to be harmonious. The Pythagorean approach is recommended here,[15] with complete enjoyment of the abrasive harmony that will result. This is not to say, however, that there aren't other valid choices: for example, if the performer were to invoke a neutral or lower E# for this lower octave, this could intensify enharmonic functional ambiguity, defensibly an artistic aim of the composer.

Please see Appendix A for a score notated with exact intonation inflections. For notes to be played in Syntonic tuning, a ratio will be derived from the lattice in Figure 4 (adjusted by key or tonicization with C# as the origin, 1:1) and a corresponding intonation instruction will be given above each note in cents above or below equal temperament, rounded to the nearest whole integer. For notes to be inflected in Pythagorean or expressive intonation, the number of cents (±) will be written above each note along with the letter P (i.e., P+10 or P-2). Note that in some cases simultaneous voices will be executing different tuning approaches: this is by design, in order to promote independence of contrapuntal voices as well as a dynamic tuning state.

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[15] The leading tone in this case is +8 cents from equal temperament adjusted for the modulation (+10 as a leading tone, -2 being based in the subdominant key)

Since a complete prose explanation of each intonation choice would be outside the scope of this paper, it is my hope that the tuning principles and intonation decisions discussed thus far will be sufficient to intrigue the reader and lead him to his own discoveries. My chief goal is only to make the reader aware of the effect that intonation can have on our organizational perception of music. The perception of varying degrees of consonance (and therefore hierarchy), a certain melodic gravity between close pitches, and—most surprisingly—form, can all be manipulated, obfuscated, or illuminated by our choices in tuning, even with the subtlest of inflections. This intonation theory is not presented as an ultimate answer to the problem of tuning, but rather as an approach to intonation that bears the same sophistication and ambiguity that we observe in the interplay of musical harmony and melody. The Beethoven case study provided demonstrates that at each instance the human must make a personal choice, and that while this choice in some cases may be flippant (in accommodation of the swaying and searching human condition), the *ramifications* of each choice are often quite specific and bound by rules of acoustics. The decision on if to interpret a passage in one key versus another will affect its tuning and overall placement in pitch space, affecting our mental map of the entire work.

It is common today for one system or approach to intonation to prevail: even unknowingly, string players generally prefer the Pythagorean system, wind players generally prefer the Syntonic system, and the predominance of the piano in our musical culture has mitigated all tuning systems often as subordinate to Equal Temperament. The hallmarks of Western tonal music, however, are far too intricate, complex, and multi-faceted for us to limit our performances of them to one approach: somehow, a composite is the answer. The particular application of that composite will reflect the nature and personality of the performer, or of the chamber ensemble. I am arguing for the perception of hierarchy in intonation—similar to the hierarchy we observe in theory analysis—and the application of only one approach to tuning is destructive to that hierarchy. In conclusion, I believe that the combined awareness of Pythagorean and Syntonic tunings, sensitivity to their possible uses for greater musical expression, and knowledge of music theory and formal analysis will generate truer artistry and success in the performance of great tonal literature.

Retuned Composite MIDI

APPENDIX A: Quatuor No. 14, op. 131

Equal Temperament MIDI

Ludwig van Beethoven


17
cresc.
cresc.
cres.
cresc.
24
A
cresc.
cresc.
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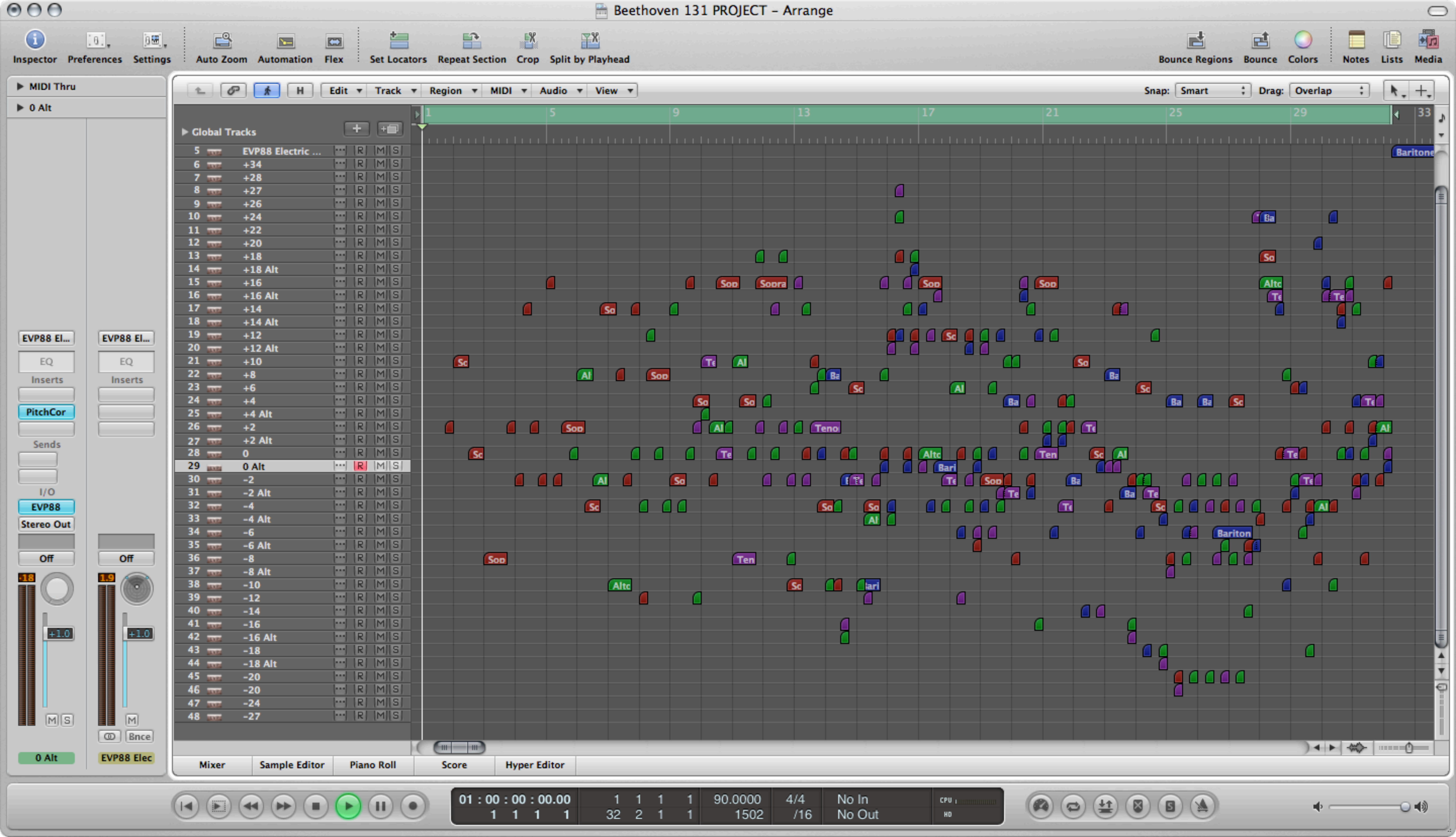
Beethoven 131 PROJECT – Arrange
Inspector
Preferences
Settings
Auto Zoom
Automation
Flex
Set Locators
Repeat Section
Crop
Split by Playhead
Bounce Regions
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13 +18
14 +18 Alt
15 +16
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17 +14
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19 +12
20 +12 Alt
21 +10
22 +8
23 +6
24 +4
25 +4 Alt
26 +2
27 +2 Alt
28 0
29 0 Alt
30 -2
31 -2 Alt
32 -4
33 -4 Alt
34 -6
35 -6 Alt
36 -8
37 -8 Alt
38 -10
39 -12
40 -14
41 -16
42 -16 Alt
43 -18
44 -18 Alt
45 -20
46 -20
47 -24
48 -27
EVP88 El...
EQ
Inserts
PitchCor
Sends
I/O
EVP88
Stereo Out
Off
Bnce
0 Alt
EVP88 Elec
Baritone
Bariton
Mixer
Sample Editor
Piano Roll
Score
Hyper Editor
01 : 00 : 00 : 00.00
90.0000
1502
4/4
/16
No In
No Out

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